

Anomalous Sudakov Form Factors

Marcello Ciafaloni

*Dipartimento di Fisica, Università di Firenze and INFN - Sezione di Firenze,
via Sansone 1, I-50019 Sesto Fiorentino, Firenze (Italy),
E-mail: ciafaloni@fi.infn.it*

Paolo Ciafaloni

*INFN - Sezione di Lecce,
Via per Arnesano, I-73100 Lecce, Italy
E-mail: paolo.ciafaloni@le.infn.it*

Denis Comelli

*INFN - Sezione di Ferrara,
Via Saragat 3, I-44100 Ferrara, Italy
E-mail: comelli@fe.infn.it*

Abstract

While radiative corrections of infrared origin normally depress high energy amplitudes (Sudakov form factors), we find that in some cases resummation of leading effects produces exponentials with positive exponents, giving rise to amplitudes that grow indefinitely with energy. The effect happens in broken gauge theories like the electroweak sector of the Standard Model, and is related to the existence of amplitudes that do not respect the gauge symmetry, and that contrary to expectations do not vanish in the very high energy limit, but rather become dominant. As a working example we consider a model with two chiral abelian gauge groups $U'(1) \otimes U(1)$ with large mass splitting $M_{Z'} \gg M_Z$, and we compute leading radiative corrections to the decay of the heavy extra Z' boson into light fermions. For proper fermionic charges, the chirality breaking magnetic dipole moment, although mass suppressed, becomes the dominant contribution to the Z' width at very high energies.

1 Introduction

The study of the asymptotic* behavior of cross sections in the Standard Model has produced a series of surprising results in recent years. In first place, this behavior is related to the infrared structure of the theory, and not to the ultraviolet one as one might naïvely assume: this is due to the fact that in the electroweak sector the symmetry breaking scale $M \sim 100$ GeV acts as an infrared cutoff producing one loop radiative corrections that grow with the c.m. energy E like $\log^2 \frac{E}{M}$ [1]. In second place, and related to this, EW radiative corrections can become huge at the TeV scale, of the order of 50% for some LHC and ILC processes [2], opening the way for the need to consider higher orders and resummation of leading effects. But the greatest surprise comes out when one tries to define "infrared free" observables, that are not affected by the above mentioned double logs: this turns out to be impossible. In fact, even if additional EW gauge bosons in the final state are included (fully inclusive observables), the cancellation between "real" and "virtual" contributions that happens in QED and QCD is spoiled by the fact that the EW symmetry is broken [3].

At this point, one might be worried by the impossibility of making perturbative predictions in the presence of radiative corrections that reduce by half the value of tree level cross sections at the TeV scale. However, our studies of resummations of leading effects show that the asymptotic behavior is theoretically well under control and can be summarized in two lines:

- All exclusive cross sections tend to zero [5].
- All inclusive cross sections tend to a linear combination of hard cross sections [3].

*by "asymptotic behavior" we mean the behavior of cross sections for energies much higher than all SM particles masses. Also the case of energies higher than the weak scale yet lower than a heavy Higgs mass have been studied, see [6]

Here an “exclusive” observable is the one usually considered in the literature: a definite final state is defined (say, two jets) and further emission of weak gauge bosons is prohibited. In the “inclusive” case, all possible emissions of weak gauge bosons in the final state is included. Since hard cross sections do not feature large logarithms (they typically correspond to tree level quantities evaluated at the relevant hard scale), the asymptotic behavior is well under control (see also [7]). The measured observables fall somewhere between the “fully exclusive” and “fully inclusive” case depending on experimental cuts; the importance of evaluating gauge bosons emissions for LHC observables has been emphasized in [8].

All the above holds in the “recovered $SU(2) \otimes U(1)$ ” limit: at energies much higher than the weak scale, the leading interactions fully respects gauge symmetry and amplitudes and cross sections obey relations dictated by the gauge symmetry: total weak isospin as well as total hypercharge are zero if we consider momenta to be all incoming. The purpose of this work is to consider amplitudes that violate these quantum numbers, and therefore vanish in the limit of unbroken gauge symmetry. On general grounds these amplitudes must be zero when the vacuum expectation value (and therefore the particles masses) go to zero, and are suppressed by powers of m/E , m being the relevant particle mass, for very high energies E . Therefore one might think that these amplitudes are negligible in the high energy limit; however, as we shall see, the dressing by soft gauge bosons can lead to surprising results.

The basic model we consider contains two chiral spontaneously broken gauge groups $U'(1) \otimes U(1)$. We assume a large mass splitting ($M \gg m_Z$, M being the Z' mass), so that the Z' -boson does not participate to the IR dynamics. Thus, the $U'(1)$ allows to construct simple amplitudes with total gauge charge violation, in our case induced by the operator $\bar{\psi} Z'_{\mu\nu} \sigma^{\mu\nu} \psi$ ($Z'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$) describing the magnetic dipole moment of the Z' gauge boson. More explicitly, since left and right fermion $U(1)$ hypercharges need not be the same, the amplitude connecting the Z' with a left fermion and a right antifermion violates $U(1)$ (hyper)charge conservation. We compute the all order ($\alpha \log^2 \frac{M^2}{m_Z^2}$)

double leading logs (DLL), taking care of the leading mass suppressed corrections of order $\mathcal{O}\left(\frac{m^2}{M^2}\right)$, m being the fermion mass. We find that, among the form factors describing the effective couplings of the Z' to the two light fermions, only the magnetic one can develop exponentially growing Sudakov like corrections.

2 The model

We start by writing the most general Lagrangian describing the gauge bosons-fermion interactions (we assume usual kinetic terms for the abelian gauge bosons):

$$\bar{\psi}_L(\partial + i g y_L Z + i g' f_L Z')\psi_L + \bar{\psi}_R(\partial + i g y_R Z + i g' f_R Z')\psi_R \quad (1)$$

where $\psi_{L/R} = \frac{1 \pm \gamma_5}{2} \psi$, $f_{L/R}$ ($y_{L/R}$) are the $U'(1)$ ($U(1)$) hypercharges for left/right fermions.

To implement, in a natural way, the spontaneous breaking of the gauge groups $U'(1) \otimes U(1)$ we need at least two complex Higgs fields, one, let's call $\phi' = \frac{1}{\sqrt{2}}(h' + v' + i \varphi')$ with v' the vev breaking $U'(1)$ and another scalar field, $\phi = \frac{1}{\sqrt{2}}(h + v + i \varphi)$ with v involved into the breaking of $U(1)$. The hierarchy $M_{Z'} \gg m_Z$ implies necessarily $v' \gg v$. The fermionic mass m being of order m_Z will be induced by the Yukawa interaction $h_f \bar{\psi}_R \phi \psi_L + h.c.$ so that $m = \frac{h_f}{\sqrt{2}} v$ and for charge conservation we need both $f_\phi = f_R - f_L = 2 f_A$ and $y_\phi = y_R - y_L = 2 y_A$.

Note that if $f_A \neq 0$ also the scalar field ϕ will participate to the breaking of $U'(1)$ and it will induce mixing between the gauge bosons $Z - Z'$ and the Goldstone modes $\varphi' - \varphi$.

In order to clarify the above considerations we write the Lagrangian for the scalar sector [†]

$$\left|(\partial_\mu + i g' f_{\phi'} Z'_\mu)\phi'\right|^2 + \left|(\partial_\mu + i g' f_\phi Z'_\mu + i g y_\phi Z_\mu)\phi\right|^2 + (h_f \phi \bar{\psi}_R \psi_L + h.c.) + V(\phi) + \mathcal{V}(\phi') \quad (2)$$

and also the gauge fixing Lagrangian (we choose to work in Feynman Gauge):

$$-\frac{1}{2}(\partial_\mu Z^\mu - g y_\phi v \varphi)^2 - \frac{1}{2}\left(\partial_\mu Z'^\mu - M \chi'\right)^2; \quad \chi' \equiv \frac{g'}{M}(f_\phi v \varphi + f_{\phi'} v' \varphi') \quad (3)$$

Working in the limit $\frac{m_{f,Z}}{M} \ll 1$ we prefer to use the gauge eigenstate basis as propagating free fields with the mass shifts used as perturbations. In this limit the propagating Z' field has mass $M^2 = g'^2 \left(f_\phi^2 v^2 + f_{\phi'}^2 v'^2\right)$, the Z field

[†]The scalar potentials $V(\phi)$ and $\mathcal{V}(\phi')$ are responsible for the generation of the spontaneous symmetry breaking scales v and v'

has mass $m_Z^2 = g^2 y_\phi^2 v^2$ and the mixing $Z' - Z$ is induced by the mass insertion $\delta M^2 = g g' f_\phi y_\phi v^2$ (note that $\frac{\delta M^2}{M^2} \ll 1$).

3 Form factors for the vertex $Z' \rightarrow \bar{f} f$

The amplitude for Z' decay $Z'_\mu(p_1 + p_2) \rightarrow \bar{f}(p_2) f(p_1)$ is given by $\varepsilon_\mu(p_1 + p_2) \bar{u}(p_1) \Gamma_\mu^{(Z')} v(p_2)$ where $\varepsilon_\mu(p)$ is the physical Z' polarization satisfying $\sum_a \varepsilon_\mu^a \varepsilon_\nu^a = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}$. In order to compute the effect induced by the multi loops generated by integrating over soft Z -gauge bosons, we introduce the more general CP invariant vertex for with $(p_1 + p_2)^2 = M^2, p_1^2 = p_2^2 = m^2$:

$$\bar{u}(p_1) \Gamma_\mu^{(Z')} v(p_2) = i g' \bar{u}(p_1) \left[\gamma_\mu (F_L P_L + F_R P_R) + \frac{m (p_1 - p_2)_\mu}{(p_1 \cdot p_2)} F_M + \frac{m (p_1 + p_2)_\mu}{(p_1 \cdot p_2)} F_P \gamma_5 \right] v(p_2) \quad (4)$$

where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$ and F_M is usually named *magnetic form factor*. We also introduce $F_V = \frac{1}{2}(F_R + F_L)$ and $F_A = \frac{1}{2}(F_R - F_L)$, the same relationships hold also for the tree level charges f_i and y_i .

Defining $\rho = \frac{m^2}{p_1 p_2}$, the amplitudes squared for the various positive (+) and negative (-) helicity fermions, summed over the Z' polarizations, are given by:

$$\frac{|\mathcal{M}_{++}|^2}{4(p_1 p_2)} = \left(F_V - F_A \sqrt{1 - \rho} \right)^2 \quad \frac{|\mathcal{M}_{--}|^2}{4(p_1 p_2)} = \left(F_V + F_A \sqrt{1 - \rho} \right)^2 \quad (5)$$

$$\frac{|\mathcal{M}_{+-}|^2}{4(p_1 p_2)} = \frac{|\mathcal{M}_{-+}|^2}{4(p_1 p_2)} = \rho \left[F_A^2 + (F_V - F_M(1 - \rho))^2 \right] \quad (6)$$

The corresponding widths can be obtained multiplying by the appropriate phase space factors. Notice that, since $(p_1 + p_2)^\mu \varepsilon_\mu(p_1 + p_2) = 0$, the form factor F_P does not contribute to physical amplitudes.

In the next section we calculate the on shell one loop form factors in the limit $M \gg m_Z, m$, retaining only the DLL contributions. Since we want to calculate the decay rates and the cross sections up to $\mathcal{O}(\rho)$, we need the values of F_M to order ρ^0 and of $F_{L,R}$ to order ρ^1 .

3.1 Form factors at One loop

Since we deal with on-shell external particles and physical polarizations, our amplitudes are gauge-invariant and can be computed in any gauge. Choosing to work in the Feynman gauge, we start analyzing carefully the one loop diagrams depicted in fig. (1). In the soft limit and at DLL approximation, we can neglect all terms proportional to the integration momentum k in the numerators, so the one loop contribution is:

$$g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{-i}{k^2 - m_Z^2} \bar{u}(p_1) \gamma_\alpha (y_L P_L + y_R P_R) \frac{p_1 + m}{2p_1 k} \Gamma_\mu^0 \frac{p_2 - m}{2p_2 k} \gamma_\alpha (y_L P_L + y_R P_R) v(p_2), \quad (7)$$

with $\Gamma_\mu^0 = i g' \gamma_\mu (f_L P_L + f_R P_R)$

and, after some basic Dirac algebra, the one loop translates in this expression capturing all the double logs $\log^2 \frac{M^2}{m_Z^2}$:

$$- (\bar{u}(p_1) J_1^\alpha \Gamma_\mu^0 J_{2\alpha} v(p_2)) \left(\int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m_Z^2} \frac{p_1 p_2}{(p_1 k)(p_2 k)} \right) \quad (8)$$

with $(y_A = \frac{y_R - y_L}{2})$

$$J_2^\mu = \frac{g}{\sqrt{p_1 p_2}} [p_2^\mu (y_L P_L + y_R P_R) - m y_A \gamma^\mu \gamma_5] \quad J_1^\mu = \frac{g}{\sqrt{p_1 p_2}} [p_1^\mu (y_L P_R + y_R P_L) + m y_A \gamma^\mu \gamma_5] \quad (9)$$

The one loop integral in double log approximation gives (recall that $M^2 = (p_1 + p_2)^2$):

$$g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m_Z^2} \frac{p_1 p_2}{(p_1 \cdot k)(p_2 \cdot k)} = g^2 \int_{m_Z^2}^{M^2} \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{p_1 p_2}{k_\perp^2} = \frac{\alpha}{4\pi} \log^2 \frac{M^2}{m_Z^2} \equiv L^2; \quad \alpha = \frac{g^2}{4\pi} \quad (10)$$

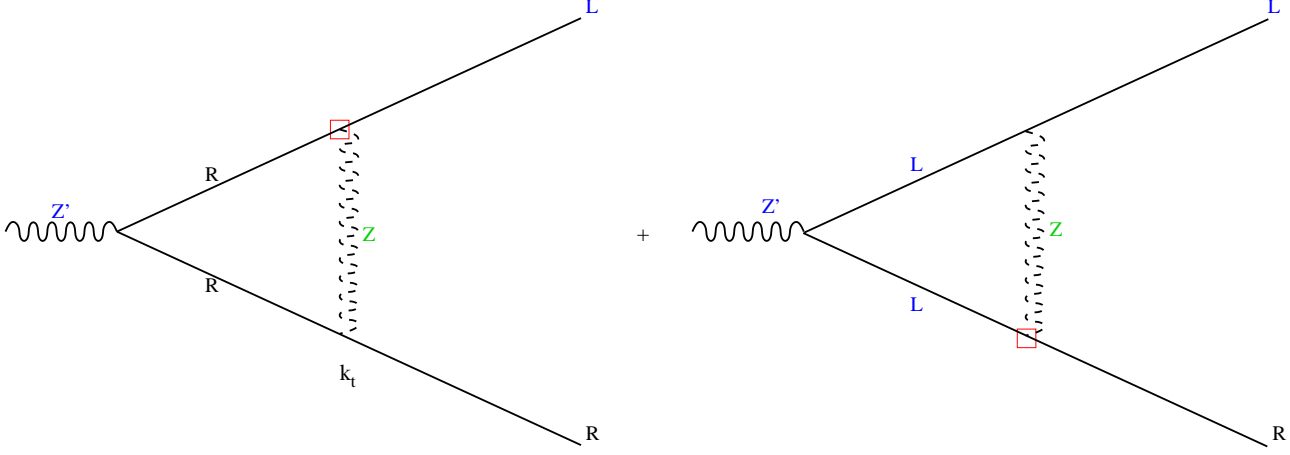


Figure 1: The anomalous magnetic moment at one loop. Diagram (a) corresponds to a mass insertion (Δ_μ) on the upper (fermion) leg and an eikonal current insertion ($J_\mu^{(eik)}$) on the lower (antifermion) leg (see eq.(13)).

where subleading single log terms have been neglected. Projecting on the different Lorentz structures, one obtains:

$$F_L^{(1)} = \left(-f_L y_L^2 + \frac{\rho}{2} f_R (y_R^2 - y_L^2) \right) L^2 \quad F_R^{(1)} = \left(-f_R y_R^2 - \frac{\rho}{2} f_L (y_L^2 - y_R^2) \right) L^2 \quad (11)$$

$$F_M^{(1)} = y_A (f_L y_L - f_R y_R) L^2 \quad F_P^{(1)} = y_A (f_L y_L + f_R y_R) L^2 \quad (12)$$

One can see that the IR double logs affect both $\mathcal{O}(\rho^0)$ and $\mathcal{O}(\rho)$ corrections; the latter are proportional to the y_A charge of the fermions, that is non zero only for chiral $U(1)$ gauge theories (clearly such double logs are not presents in QED [9] and QCD).

It is instructive and useful for the next section to separate the currents J_α^i defined in (9) in two components, the first one being the usual eikonal current (J_{eik}^μ) and the second one (Δ^μ) responsible of the chirality flip, proportional to the fermion mass m insertion

$$J_2^\mu = J_{\text{eik}}^\mu(p_2) - \Delta^\mu, \quad J_1^\mu = J_{\text{eik}}^\mu(p_1) + \Delta^\mu, \quad \Delta^\mu \equiv \frac{g}{\sqrt{p_1 p_2}} m y_A \gamma^\mu \gamma_5 \quad (13)$$

Let us now discuss another class of diagrams coming from $Z - Z'$ mixing and from the Higgs/Goldstone sector (see fig. (2)).

- Mixing effects in $Z' - Z$ sector (see the diagram to the left of fig.(2)) simply produce a shift $f_{L,R} \rightarrow f_{L,R}^0 = f_{L,R} + \frac{g}{g'} \frac{\delta M^2}{M^2} y_{L,R}$ so that eqs. (11, 12) are still valid with the replacement $f_{L,R} \rightarrow f_{L,R}^0$; the same holds for the all order resummed expressions discussed in next section. In other words, $Z - Z'$ mixing only induces a (small $\mathcal{O}(\frac{m_Z^2}{M^2})$) renormalization of the $U(1)$ fermion hypercharges.

- The evaluation of diagram a) gives [‡]

$$F_L^{(a)} = \frac{h_f^2}{16\pi^2} \rho f_L \log^2 \frac{M^2}{m_h^2} \quad F_R^{(a)} = \frac{h_f^2}{16\pi^2} \rho f_R \log^2 \frac{M^2}{m_h^2} \quad F_{M,P}^{(a)} = 0. \quad (14)$$

- The corrections from the diagram b) include two different contributions where the role of h and φ are interchanged; these two diagrams have opposite signs and cancel out completely at the DLL level.
- From diagrams c)+d) we get

$$F_L^{(c+d)} = -\frac{\alpha}{\pi} \rho f_\phi y_\phi y_L \log^2 \frac{M^2}{m_{Z,h}^2} \quad F_R^{(c+d)} = -\frac{\alpha}{\pi} \rho f_\phi y_\phi y_R \log^2 \frac{M^2}{m_{Z,h}^2} \quad F_{M,P}^{(c+d)} = 0. \quad (15)$$

the IR cutoff $m_{Z,h}$ is a mixing of the gauge and Higgs masses depending on their relative magnitude (see [4]).

[‡]Note that only the scalar h , and not the pseudoscalar φ , contributes at the DLL level.

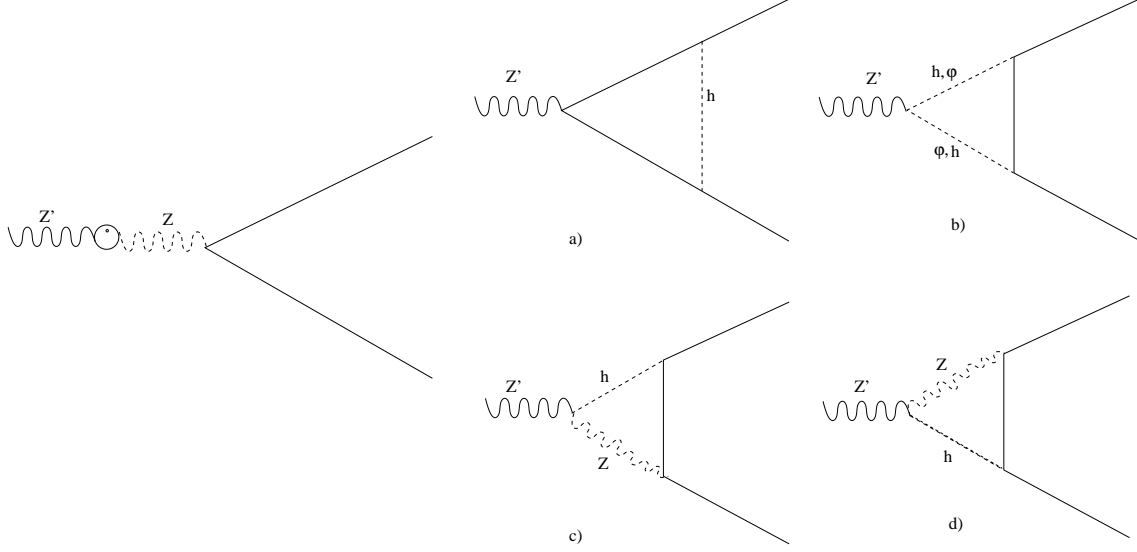


Figure 2: Mixing and scalar loop effects to $Z' \rightarrow \bar{\psi}\psi$. To the left we show the $Z' - Z$ mixing corrections. To the right we have the one loop scalar corrections : straight dotted line are the light scalars h and ϕ while wavy lines are light Z gauge bosons.

4 All order resummed form factors

Since we work in the regime $M \gg m_Z$, our first order calculations cannot be trusted because $L^2 \gg 1$, and we have to proceed to the resummation of all the DLL (L^{2n}).

The dressing by soft boson insertions of the eikonal type can be explicitly taken into account at all orders by making use of the eikonal identity (see [10] for instance). We illustrate this calculation by adopting the method of k_\perp -ordering: the leading terms in the resummed series are given by “ladder” insertions ordered in the soft variable k_\perp , which is the transverse momentum of the soft gauge boson[§].

The resummation of the soft gauge bosons for momenta in the range $k_\perp^{inf} \leq k_\perp \leq k_\perp^{sup}$ is given by the following Sudakov form factor [11]:

$$S_{i,j}[k_\perp^{sup}, k_\perp^{inf}] = \exp \left[-\frac{\alpha}{2\pi} y_i y_j \int_{k_\perp^{inf}}^{k_\perp^{sup}} \frac{dk_\perp^2}{k_\perp^2} \log \frac{M^2}{k_\perp^2} \right] = \exp \left[-\frac{\alpha}{4\pi} y_i y_j \left(\log^2 \frac{M^2}{k_\perp^{2 inf}} - \log^2 \frac{M^2}{k_\perp^{2 sup}} \right) \right] \quad (16)$$

where y_i, y_j are the relevant $U(1)$ charges.

In general terms we have only three possible Sudakov structures: S_{LL} , S_{RR} and $S_{L,R}$ that after momenta integration will generate three kind of Sudakov exponents: $e^{-y_L^2 L^2}$, $e^{-y_R^2 L^2}$ and $e^{-y_L y_R L^2}$. While the first two are exponentially suppressing their multiplicative factors, the last one ($e^{-y_L y_R L^2}$), that we call Anomalous Sudakov, depending on the sign of the charges $y_{L,R}$ can generate exponential growing corrections (for $y_L y_R < 0$).

In fig. (3) we schematically show the dressing of a one loop result, with multiple insertions of soft gauge bosons. In fig. (3) *b* the red boxes represent the “cloud” of all-order resummed DLL soft gauge bosons, while the wavy line is an insertion with a chirality-flip vertex (Δ_ν) on the upper fermion leg and an eikonal insertion (J_{eik}^ν) on the lower (antifermion) leg. The red box to the right represents “very soft” gauge bosons with transverse momenta k'_\perp satisfying $m_Z < k'_\perp < k_\perp$, k_\perp being the wavy line Z momentum; bosons in the red box to the left satisfy $k_\perp < k'_\perp < M$.

In order to get the all order DLL amplitudes we show how the various one loop terms are dressed by the all order corrections:

- The double insertion of the two eikonal currents at one loop is replaced by the usual DLL Sudakov Form Factors

[§]we have checked that the explicit computation using the eikonal identity produces the same results obtained by k_\perp -ordering

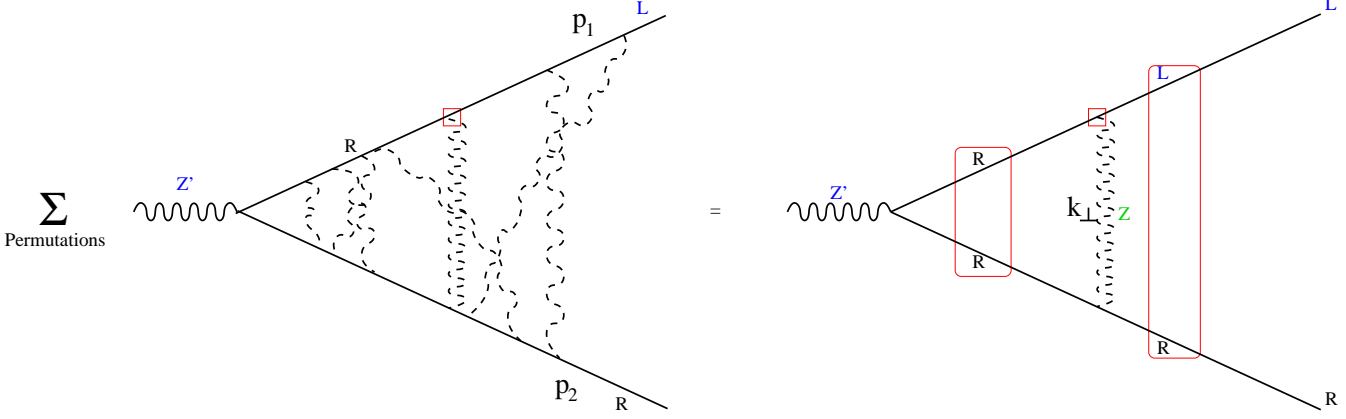


Figure 3: All order Chirality breaking amplitude for the process $Z' \rightarrow \bar{f}_R f_L$ mediated by the Z bosons (dashed lines). For simplicity the Z' couples only to right fermions. To the left we draw the general structure of all the Feynman diagrams that we have to sum up, to the right we used the ladder diagram approach based on soft k_\perp ordering where the red blocks means Sudakov form factors (see eq. (16)).

for $F_{L,R}$:

$$- J_\alpha^{\text{eik}} \Gamma_\mu^0 J_\alpha^{\text{eik}} \int_{m_Z^2}^{M^2} \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{M^2}{k_\perp^2} \rightarrow \gamma_\mu (S_{RR}[M, m_Z] P_R + S_{LL}[M, m_Z] P_L) \quad (17)$$

and the corresponding dressed form factors are

$$F_L^{(J^2)}(s) = f_L S_{LL}[M, m_Z] = f_L e^{-y_L^2 L^2} \quad F_R^{(J^2)}(s) = f_R S_{RR}[M, m_Z] = f_R e^{-y_R^2 L^2} \quad (18)$$

These terms represent the "usual" Sudakov corrections coming from the DLL interactions that do not break the gauge symmetries.

- The one loop diagram with one eikonal current on one leg and one chirality-flip current Δ_μ on the other is dressed by soft Z insertions (see fig.3). This generates the "anomalous" $F_{M,P}$ form factors plus an extra contribution to $F_{R,L}$:

$$\Delta_\alpha \Gamma_\mu^0 J_\alpha^{\text{eik}} \int \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{M^2}{k_\perp^2} \rightarrow g^2 y_A \frac{m \not{p}_2}{(p_1 p_2)} \gamma_\mu \int \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \left(-f_R y_R S_{RR}[M, k_\perp] \log \frac{M^2}{k_\perp^2} S_{LR}[k_\perp, m_Z] P_R \right. \\ \left. + f_L y_L S_{LL}[M, k_\perp] \log \frac{M^2}{k_\perp^2} S_{LR}[k_\perp, m_Z] P_L \right) \quad (19)$$

$$J_\alpha^{\text{eik}} \Gamma_\mu^0 \Delta_\alpha \int \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{M^2}{k_\perp^2} \rightarrow g^2 y_A \gamma_\mu \frac{m \not{p}_1}{(p_1 p_2)} \int \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \left(-f_R y_R S_{RR}[M, k_\perp] \log \frac{M^2}{k_\perp^2} S_{LR}[k_\perp, m_Z] P_L \right. \\ \left. + f_L y_L S_{LL}[M, k_\perp] \log \frac{M^2}{k_\perp^2} S_{LR}[k_\perp, m_Z] P_R \right) \quad (20)$$

The above contributions, sandwiched between the spinors $\bar{u}(p_1)$ and $v(p_2)$ produce the "anomalous" form factors

$$F_M^{(J\Delta)} \equiv F_M = \frac{1}{2} (f_L e^{-y_L^2 L^2} + f_R e^{-y_R^2 L^2}) - \frac{1}{2} (f_L + f_R) e^{-y_L y_R L^2} \quad (21) \\ F_P^{(J\Delta)} \equiv F_P = \frac{1}{2} (f_L e^{-y_L^2 L^2} - f_R e^{-y_R^2 L^2}) - \frac{(f_L - f_R)}{2} e^{-y_L y_R L^2}$$

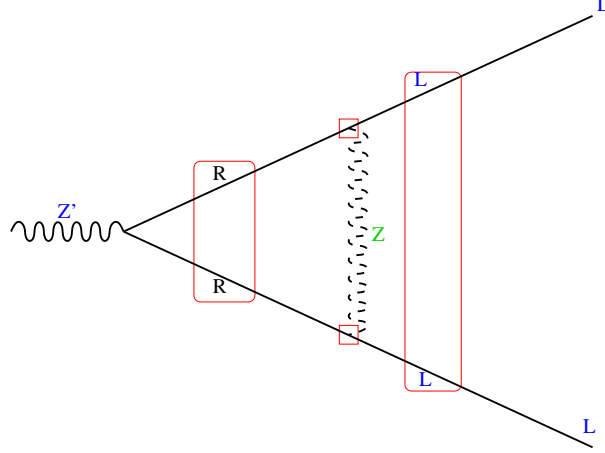


Figure 4: All order Chirality breaking amplitude for the process $Z' \rightarrow \bar{f}_L f_L$ mediated by $(\Delta_\mu)^2$ insertions.

and the corrections to $F_{L,R}$

$$F_L^{(J\Delta)} = -\rho f_R \left(e^{-y_L y_R L^2} - e^{-y_R^2 L^2} \right) \quad F_R^{(J\Delta)} = -\rho f_L \left(e^{-y_L y_R L^2} - e^{-y_L^2 L^2} \right) \quad (22)$$

- Finally the soft DLL gauge bosons dressing the one loop diagram with two Δ_μ currents generates a contribution to $F_{R,L}$ (see fig.4):

$$-\Delta_\alpha \Gamma_\mu^0 \Delta_\alpha \int_{m_Z^2}^{M^2} \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{M^2}{k_\perp^2} \rightarrow 2m^2 \frac{g^2}{(p_1 p_2)} y_A^2 \gamma_\mu \int \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} (f_L S_{LL}[M, k_\perp] S_{RR}[k_\perp, m_Z] P_R + f_R S_{RR}[M, k_\perp] S_{LL}[k_\perp, m_Z] P_L) \log \frac{M^2}{k_\perp^2} \quad (23)$$

$$F_L^{(\Delta^2)}(s) = 2 \frac{\rho f_R y_A^2}{(y_L^2 - y_R^2)} \left(e^{-y_R^2 L^2} - e^{y_L^2 L^2} \right) \quad F_R^{(\Delta^2)} = 2 \frac{\rho f_L y_A^2}{(y_L^2 - y_R^2)} \left(e^{-y_R^2 L^2} - e^{y_L^2 L^2} \right)$$

The resummed contribution to $F_{L,R}$ in eq. (22) features an “anomalous” exponent proportional to $-y_L y_R$, that can be positive for left and right hypercharges of opposite signs. However this contribution, as we show now, is canceled by a contribution from the DLL of the two loop diagram in fig. 5, that also give $\mathcal{O}(\rho)$ corrections to $F_{L,R}$.

The two loop expression is given by:

$$2 m^2 \frac{g^2}{Q^4} y_A^2 y_L y_R \not{p}_2 \gamma_\mu \not{p}_1 (f_L P_R + f_R P_L) \int_{m_Z^2}^{M^2} \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{M^2}{k_\perp^2} \int_{k_\perp^2}^{M^2} \frac{dq_\perp^2}{8\pi^2} \frac{1}{q_\perp^2} \log \frac{M^2}{q_\perp^2} \quad (24)$$

while in order to evaluate the insertion of DLL soft gauge boson we have to evaluate the integral

$$2 m^2 \frac{g^2}{Q^4} y_A^2 y_L y_R \not{p}_2 \gamma_\mu \not{p}_1 \int_{m_Z^2}^{M^2} \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \int_{k_\perp^2}^{M^2} \frac{dq_\perp^2}{8\pi^2} \frac{1}{q_\perp^2} \log \frac{M^2}{k_\perp^2} \log \frac{M^2}{q_\perp^2} (f_L S_{LL}[M, k_\perp] S_{LR}[k_\perp, q_\perp] S_{RR}[q_\perp, m_Z] P_R + f_R S_{RR}[M, k_\perp] S_{LR}[k_\perp, q_\perp] S_{LL}[q_\perp, m_Z] P_L) \quad (25)$$

The result for the leading $\mathcal{O}(\rho)$ comes from the substitution $\not{p}_2 \gamma_\mu \not{p}_1 \rightarrow -2(p_1 p_2) \gamma_\mu$:

$$F_L^{(J\Delta)^2} = \rho f_R \left(e^{-y_L y_R L^2} - \frac{1}{(y_L + y_R)} \left(y_L e^{-y_R^2 L^2} + y_R e^{-y_L^2 L^2} \right) \right) \quad (26)$$

$$F_R^{(J\Delta)^2} = \rho f_L \left(e^{-y_L y_R L^2} - \frac{1}{(y_L + y_R)} \left(y_L e^{-y_R^2 L^2} + y_R e^{-y_L^2 L^2} \right) \right)$$

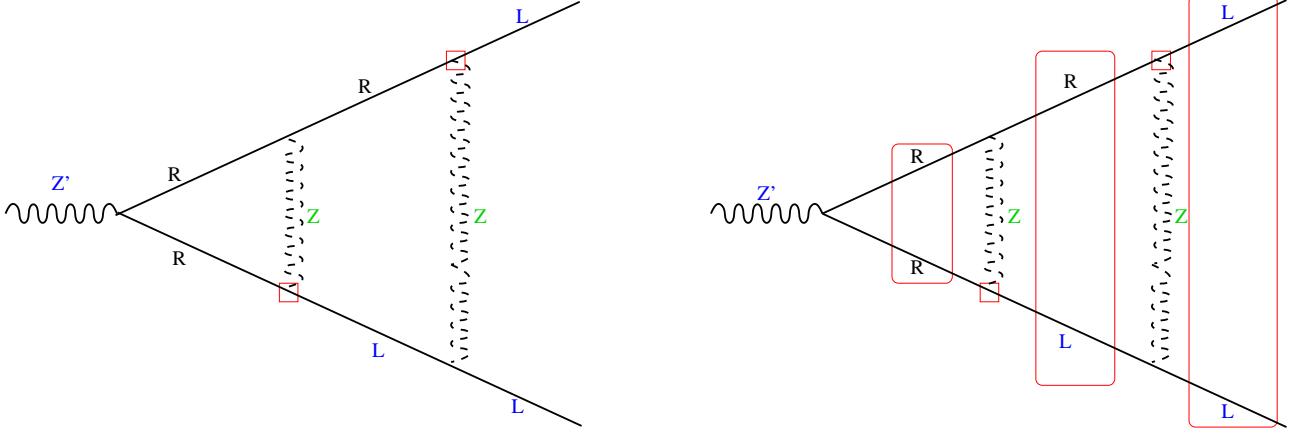


Figure 5: The two loop (to the left) and the all order (to the right) Chirality breaking amplitudes for the process $Z' \rightarrow \bar{f}_L f_L$ mediated by $(\Delta_\mu J_\nu^{(eik)})^2$ insertions.

Finally adding all together (eqs.(18,21,22,26)) we obtain the following results coming from pure Z boson exchanges:

$$F_L^{(Z)} = f_L e^{-y_L^2 L^2} - \frac{\rho}{2} f_R (e^{-y_R^2 L^2} - e^{-y_L^2 L^2}) \quad (27)$$

$$F_R^{(Z)} = f_R e^{-y_R^2 L^2} - \frac{\rho}{2} f_L (e^{-y_L^2 L^2} - e^{-y_R^2 L^2}) \quad (28)$$

$$F_M^{(Z)} = \frac{1}{2}(f_L e^{-y_L^2 L^2} + f_R e^{-y_R^2 L^2}) - \frac{1}{2}(f_L + f_R) e^{-y_L y_R L^2} \quad (29)$$

$$F_P^{(Z)} = \frac{1}{2}(f_L e^{-y_L^2 L^2} - f_R e^{-y_R^2 L^2}) - \frac{1}{2}(f_L - f_R) e^{-y_L y_R L^2} \quad (30)$$

Let us now consider the all order dressing of the diagrams appearing in fig. 2. As we are going to show now, these diagrams cannot produce the “anomalous” effects we are studying here.

- The mixing effects in $Z' - Z$ sector simply amount to a renormalization of the couplings; such renormalization is unphysical, as discussed previously
- The soft gauge boson cloud for the Higgs boson exchange of fig.(2) , diagram a), gives (for $m_h = m_Z$):

$$F_L^{(a)} = \frac{h_f^2}{16\pi^2} \rho f_L \log^2 \frac{M^2}{m_Z^2} e^{-y_L^2 L^2} \quad F_R^{(a)} = \frac{h_f^2}{16\pi^2} \rho f_R \log^2 \frac{M^2}{m_Z^2} e^{-y_R^2 L^2} \quad (31)$$

- For the scalar loop exchange we have to dress only the diagrams c)+d) of fig.(2). In this case the dressing factor (16) is acting only on the external legs giving

$$F_L^{(c+d)} \equiv F_L^{hZ} = -4 g^2 f_\phi y_\phi y_L \rho \int \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{M^2}{k_\perp^2} S_{LL}[k_\perp, m_Z] = 4 f_\phi \frac{y_\phi}{y_L} \rho (e^{-y_L^2 L^2} - 1) \quad (32)$$

$$F_R^{(c+d)} \equiv F_R^{hZ} = -4 g^2 f_\phi y_\phi y_R \rho \int \frac{dk_\perp^2}{8\pi^2} \frac{1}{k_\perp^2} \log \frac{M^2}{k_\perp^2} S_{RR}[k_\perp, m_Z] = 4 f_\phi \frac{y_\phi}{y_R} \rho (e^{-y_R^2 L^2} - 1) \quad (33)$$

where we taken $m_h = m_Z$ for convenience.

Our conclusions about the terms generated by $Z' - Z$ mixing and the Goldstone/Higgs sector are therefore the following:

- Including $Z' - Z$ mixing only produces an unphysical (small) renormalization of the couplings.
- Terms produced by the Goldstone/Higgs sector only affect the vector and axial form factors and are depressed at high energy by “standard” ($e^{-y_L^2 L^2}$, $e^{-y_R^2 L^2}$) Sudakov form factors

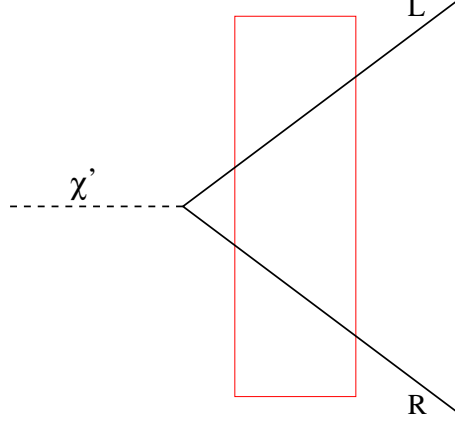


Figure 6: All order DLL amplitude for the Goldstone amplitude $\chi' \rightarrow \bar{\psi} \psi$.

- All of these effects are unrelated to the ones produced by Z exchange since they are written in terms of independent parameters of the theory f_ϕ, h_f, m_h and they vanish in some limit ($h_f \rightarrow 0, f_\phi \rightarrow 0$, heavy Higgs)

In order to obtain a cross check of our results we can use Ward Identities (WI) that connect the amplitude $\Gamma_{Z' \bar{f} f}^\mu$ with the amplitude $\Gamma_{\chi' \bar{f} f}$, χ' being the Goldstone boson of the Z' (see eq.(3)).

$$\chi' = \frac{g'}{M} (f_{\phi'} v' \varphi' + f_\phi v \varphi) \quad (34)$$

The Goldstone χ' will interact with the fermions with coupling

$$2 f_A g' \frac{m}{M} \chi' \bar{u}(p_1) \gamma_5 v(p_2) \quad \xrightarrow[\text{at all orders}]{} \quad 2 F_{\chi'} g' \frac{m}{M} \chi' \bar{u}(p_1) \gamma_5 v(p_2) \quad (35)$$

where $F_{\chi'}$ is the all orders form factor for the operator $\chi' \bar{\psi} \gamma_5 \psi$.

The relevant WI reads :

$$(p_1 + p_2)_\mu \Gamma_{Z' \bar{f} f}^\mu = M \Gamma_{\chi' \bar{f} f} \quad \text{or} \quad F_A + (1 + \rho) F_P = F_{\chi'} \quad (36)$$

that at tree level ($F_A^{(0)} = f_A, F_P^{(0)} = 0, F_{\chi'}^{(0)} = f_A$) is trivially satisfied. When we introduce the soft cloud (at order $\mathcal{O}(\rho^0)$) we get the first non trivial consistency relation after we explicit evaluate $F_{\chi'}$ at all orders in L and at order $\mathcal{O}(\rho^0)$ (see fig. 5). Evaluating the Feynman diagram depicted in fig.5, with a simple calculation we obtain

$$f_{\chi'} \rightarrow F_{\chi'} = f_A S_{LR}[M, m_Z] = f_A e^{-y_L y_R L^2} \quad (37)$$

The check of eq.(36) at order $\mathcal{O}(\rho)$ requires a calculation of $\mathcal{O}(\rho^2)$ (due to the fact that we need F_P at order $\mathcal{O}(\rho)$) that is beyond the present purposes.

Overall, we can summarize the results of this section in the following way:

- the axial and vector form factors related to F_L and F_R receive, after resummation, only “standard” Sudakov form factors ($e^{-y_L^2 L^2}, e^{-y_R^2 L^2}$) that exponentially suppress the amplitudes at very large energies.
- The magnetic dipole moment form factor F_M gets dressed also with the *Anomalous Sudakov* ($e^{-y_L y_R L^2}$) whose exponent can be *positive* if $y_L y_R < 0$. If this is the case, F_M asymptotically dominates over F_L, F_R .

5 Asymptotic dynamics

If $y_L y_R < 0$, the terms proportional to the exponentially growing form factor F_M in the squared amplitudes (5,6) dominate over the terms in $F_{L,R}$ for $M \gg m_z, m$. At what energy scales M does this happens ?

Let us consider, for simplicity, a vector like Z' where $f_{L,R} \equiv f$ (in this case $f_A = 0$ and all the mixing terms and scalar loops disappear). The helicity changing decay rate Γ_{+-} becomes:

$$\Gamma_{+-} \simeq \Gamma_{+-}^0 \frac{1}{4} \left(4 e^{-2y_L y_R L^2} + e^{-2y_R^2 L^2} - 2 e^{-(y_L^2 + y_R^2) L^2} + e^{-2y_L^2 L^2} \right) + \mathcal{O}(\rho^2) \quad (38)$$

where Γ_{+-}^0 is the tree level rate. The resummed expression is a combination of decreasing and one potentially increasing (for $y_L y_R < 0$) exponentials. In the limit $L^2 \gg 1$ and for $y_L y_R < 0$ quickly the resummed value becomes twice as big as the tree level one, giving a 100 % radiative correction that puts in evidence the importance of the resummation. This happens for scales such that:

$$e^{-2y_L y_R L^2} = 2 \Rightarrow \frac{M}{m_Z} = \exp \left[\sqrt{\frac{\pi \log 2}{-2y_L y_R \alpha}} \right] \quad (39)$$

For $y_L = -y_R = 1$, $\alpha \sim 1/30$ and $m_Z \sim 100$ GeV one obtains energies of the order of 30 TeV, which is a relatively low scale value!

For other observable like the full decay rate $\bar{\Gamma}$ the expansion in ρ gives (always taking $f_L = f_R = f$)

$$\bar{\Gamma} \propto f^2 (e^{-2y_R^2 L^2} + e^{-2y_L^2 L^2}) + \rho f^2 \left(2e^{-2y_L y_R L^2} + e^{-2y_R^2 L^2} - 2e^{-(y_L^2 + y_R^2) L^2} + e^{-2y_L^2 L^2} \right) + \mathcal{O}(\rho^2) \quad (40)$$

In this case the anomalous Sudakov is always multiplied by a power of ρ .

If we compare the $\rho = 0$ terms with the anomalous exponential corrections, we see that they are of the same order when

$$\rho e^{-2y_L y_R L^2} \sim e^{-2y_R^2 L^2} \quad (41)$$

and for $m \sim m_Z$ (just to have the order of magnitude) this happens at mass scales

$$M \sim m e^{\frac{2\pi}{\alpha(y_{L,R}^2 - y_L y_R)}} \quad (42)$$

that is of the same order of the Landau Pole (LP) energy $E_{LP} \sim m e^{\frac{\pi}{\beta\alpha}}$ (where β is the beta function of the $U(1)$ gauge group).

Are these effects present also into the SM?

It is straightforward to identify the chiral gauge group $U(1)$ with $U(1)_Y$ with m_Z exactly the gauge boson Z mass of 91 GeV. Then, from the analysis of the quantum number of the SM fields we see that $U(1)$ “anomalous” Sudakov form factors are presents only for the down quark sector where $y_L = \frac{1}{6}$ and $y_R = -\frac{1}{3}$ so that $y_L y_R = -\frac{1}{18} < 0$.

The phenomenological relevance of the above effects in this case result quite suppressed first of all for the smallness of the gauge coupling $\alpha_Y \sim \frac{1}{60}$ and secondarily also for the smallness of the charges $y_L y_R = -\frac{1}{18}$.

The presence of anomalous Sudakov for the non abelian $SU(2)$ part is at present under study and results quite interesting because we naively expect phenomenological relevant effects already at TeV scale (see eq. (39)) mainly due to the fact that the gauge coupling is large ($\alpha_W \sim 2\alpha_Y$) and the the non abelian charges are naturally $\mathcal{O}(1)$ [12].

6 Conclusions

In this work we have evaluated the form factors of a very heavy Z' gauge boson of mass M into a fermion-antifermion pair in a simple $U(1) \otimes U'(1)$ model, performing the calculation up to order m^2 in the fermion mass m and to all orders in the $U(1)$ gauge coupling at the double log level $(\alpha \log^2 \frac{M^2}{m_Z^2})^n$. We conclude that while the axial and vector form factors feature a “standard”, energy decreasing Sudakov form factor, the magnetic dipole moment features an “anomalous” exponential $\sim \exp[-\alpha y_L y_R \log^2 \frac{M^2}{m_Z^2}]$ term, which grows with energy for fermions having opposite left-right $U(1)$ charges ($y_L y_R < 0$). This feature belongs exclusively to broken gauge theories like the electroweak sector

of the Standard Model, and is a most unusual one. In fact the magnetic dipole moment corresponds to the insertion of an effective dimension five operator of the form $\bar{\psi}_L \mathcal{Z}'_{\mu\nu} \sigma^{\mu\nu} \psi_R$ ($\mathcal{Z}'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$), which explicitly breaks $U(1)$ if $y_L \neq y_R$ and is (must be) proportional to the $U(1)$ vacuum expectation value. The expectation is that at large energy scales, where symmetry is recovered, this symmetry violating operator gives negligible contribution to observables: this is by no means the case. While the contribution is truly suppressed by fermion masses at tree level, the dressing by IR dynamics around the light Z mass makes this operator the leading one at very high energies. This is a kind of “non decoupling” in the sense that very high energies observables are sensitive to the very low IR cutoff scale, whatever the ratio of the scales. This is due to the high energy behavior being dictated by the IR dynamics, and therefore sensitive to symmetry breaking at *any* scale.

The main qualitative difference with respect to all the previous Sudakov form factor evaluations (in QED and in QCD but also in the high energy EW sector [5]) is the fact that the amplitudes we consider here do not conserve the gauge $U(1)$ charge of the soft Z gauge bosons. In QED for photons and in QCD for gluons such conservation is automatic because the gauge symmetries are exact, while in the EW case up till now only the leading operators where no $SU(2)$ or $U(1)_Y$ breaking is involved have been considered. The magnetic dipole moment instead is proportional to the chirality flip operator $\bar{\psi}_L \psi_R$ and from the $U(1)$ point of view carries a net $y_L - y_R$ charge: this is the main reason that allows for the presence of unusual behavior in the Sudakov form factors. As a result, the helicity flip width Γ_{+-} can grow indefinitely at large energies.

A number of questions naturally arise: What is the role played by Z emission corrections? How the cancellation theorems [13] involving real and virtual corrections work? What happens in the Standard Model itself, where the non abelian nature could significantly change things? These problems will be addressed in future investigations.

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